3 Chapter 3 -- Returns on Alternate Savings Vehicle: In this Chapter, we will look at savings vehicles that return the same pre-tax return but differ in their tax treatments to the investor. Note that the assumption that all savings vehicles return the same pre-tax return is unrealistic unless the counterparty faces the same tax treatment regardless of the form of the investment. Furthermore, if one savings vehicle is taxed more preferentially than another, over time the cost of the preferentially-tax vehicle will be bid up as compared with other investments, eventually equalizing the returns once implicit taxes as well as explicit taxes are taken into account. Nevertheless, in this chapter we will ignore such market forces, varying only the tax consequences of the acquisition (deductible or not), the timing of taxation (periodic, at disposition, or never), and rate (ordinary income or capital gain).

3.1 Intertemporally Constant Tax Rates

3.1.1 Review of Compound Interest: Assume that $I is invested in a savings vehicle growing at the rate of R per year and that income is taxed at a rate of t.

3.1.1.1 The pre-tax return after n years equals $\mathbf{\$I(1 + R)^n}$.

3.1.1.2 If the return is taxed each year, the after-tax return is $\mathbf{\$I[1 + R(1 - t)]^n}$.

3.1.1.3 If the return is taxed only at disposition, then the after-tax return equals $\mathbf{\$I(1 + R)^n - t\$I(1 + R)^n + t\$I = \$I(1 + R)^n(1 - t) + t\$I.}$

3.1.2 Investment in Savings Vehicles I and II

3.1.2.1 Savings vehicle I is the base-line: the investment is not deductible and the return is taxed periodically at ordinary rates. The after-tax rate of return is thus R(1 – t). For example, if R = 7% and t = 30%, then the after-tax rate of return equals 4.9%.

3.1.2.2 Savings vehicle II describes exploitation of the realization doctrine without the benefit of any tax rate arbitrage; that is, the investment is not deductible and the income is taxed only at disposition at ordinary rates. If the investment horizon is one year, there is no difference between this vehicle and vehicle I. But as the investment horizon lengthens, the after-tax rate of return is asymptotic to R. Note that while the after-tax rate of return approaches the pre-tax rate of return, the dollar difference between an exempt return and vehicle II is significant. See the text in the second paragraph on page 3-6.

3.1.3 Hybrid Savings Vehicles: Some investments represent a combination of vehicles I and II. Convertible debt, for example, pays a periodic return but also includes a growth component protected by the realization doctrine.

3.1.4 Investments in Savings Vehicles III: Vehicle III is identical to Vehicle I except the periodic tax is imposed at the preferential capital gains rate. Such investment vehicles are rare.

3.1.5 Investments in Savings Vehicle IV: Vehicle IV is identical to vehicle II except the tax at disposition is imposed at the preferential capital gains rate. While vehicle II is common for corporate investments, vehicle IV is common by noncorporate taxpayers.

3.1.6 Investment in Savings Vehicle V: Vehicle V includes 529 college plans, ROTH IRAs, and most life insurance policies. The after-tax return equals the pre-tax return.

3.1.7 Investment in Savings Vehicle VI: To compensate for the immediate deduction, the taxpayer can make a greater pre-tax investment (going from $I to $I/(1-t)) without making a greater post-tax investment (this assumes the investment can scale). As a result, the return equals **$[\$I/(1 – t)](1 + R)^n(1 – t)$**, or **$\$I(1 + R)^n$**. Thus, *when tax rates are constant over time, vehicles V and VI produce the same returns*.

3.2 Changes in Tax Rates Over Time: How do the vehicles compare if tax rates are expected to change over time? If rates are expected to decrease, then deferred taxation offers the double benefit of deferral and exploiting the reduced rates. Note that if tax rates are expected to rise, then deferred taxation offers deferral but at the cost of *negative* tax arbitrage. Over long investment horizons, the deferral will dominate the change in tax rates but over short horizons either effect can dominate.

3.3 More on Pension Plans

3.3.1 Traditional Deductible Plans: As of 2019, compensation up to $6,000 ($7,000 if age 50 or older) may be contributed to a traditional IRA. The contribution is deductible, the investment accumulates tax-free, and the return usually is received as an annuity taxable as ordinary income. Note that the dollar contribution limitations are imposed on the *pre-tax* contributions. A traditional IRA is a type VI vehicle.

3.3.2 Nondeductible IRAs: Under some circumstances, a taxpayer may make a nondeductible contribution to a traditional IRA.

3.3.3 Roth IRAs: Contributions are nondeductible, the investment accumulates tax-free, and the return also is tax-free. Roth IRAs have the same contribution limits as traditional IRAs but these limits are now imposed on a post-tax basis. Roth IRAs are a type V vehicle. Recall from paragraph 3.1.7 above that, if tax rates are constant, vehicles V and VI produce the same returns.

3.3.4 Comparison of the Deductible and Roth IRAs: New Contributions -- As discussed on page 3-15 (equation 3.9), the difference between the return to a Roth IRA and a traditional IRA can be represented as: $\$I(1 + R)^n[1 – (1 – t_n)/(1 – t_o)]$. *Note that this assumes equal post-tax investments*; that is, $\$I/(1 – t_o)$ must be invested in the traditional IRA. Because each type of IRA has an annual contribution limit of $6,000 ($7,000 if age 50 or over), an investor cannot necessarily equalize contributions between a traditional IRA and a Roth IRA. For example, if the taxpayer is in the 30% tax bracket, then a $6,000 contribution to a Roth IRA is equivalent to a $8,571 (i.e., $6,000/0.70) investment in a traditional IRA. A taxpayer could make a $6,000 contribution to a traditional IRA and a $2,571 investment in some alternate vehicle, but that alternative vehicle is unlikely to produce a yield equivalent to an IRA, making the Roth IRA a better investment. For example, a maximum contribution to a Roth IRA dominates a

maximum contribution to a traditional IRA coupled with an investment of t$Max/(1 – t) in a vehicle II product. If tax rates decline over time, then the advantage of the Roth IRA may not be sufficient to compensate for losing the lower rate. See Table 3.5 at page 3-17.

3.3.5 Comparison of the Deductible and Roth IRAs: The Conversion Decision -- Accumulations in traditional IRAs can be rolled over into Roth IRAs by the payment of tax at ordinary rates on the value of the roll-over amount. If you think of the roll-over amount as taxable income, then electing not to roll the investment into a Roth IRA is equivalent to placing the amount in a traditional IRA. Thus, a roll-over should be preferred only if a new contribution to a Roth IRA would be preferred to a new contribution to a traditional IRA. *The wrinkle is that taxes due on the roll-over can be paid out of non-pension funds, effectively increasing the amount of the post-tax Roth IRA contribution.*

3.3.5.1 Suppose, for example, that Taxpayer has $200,000 invested in a traditional IRA eligible for roll over into a Roth IRA. Taxpayer also has savings of $80,000, and Taxpayer is in the 40% tax bracket. Assume tax rates will not change. How much will Taxpayer have after 10 years if Taxpayer elects the roll-over and uses the savings to pay the roll-over tax? How much will Taxpayer have if Taxpayer leaves the pension as a traditional IRA and invests the $80,000 in a type II vehicle? In each case, assume an annual pre-tax return of 7% and an investment horizon of 10 years? $200,000*(1.07)^{10}, or $393,430 vs. $200,000*(1.07)^{10}*(0.60) (i.e., $236,058) + $80,000(1.07)^{10}*(0.60) +0.4*$80,000 (or $94,423 + $32,000) = $362,481.

3.3.5.2 Reconsider this example, assuming that Taxpayer elects to pay the roll-over taxes out of the IRA funds. How does this change the results? The first issue is whether the roll-over tax is a *net tax* or a *gross tax*. If it is a *net tax*, then the tax (assumed to be at 40% in this problem), is imposed on the amount transferred into the Roth IRA. If the taxpayer rolls-over $142,857, then the tax on that amount equals 0.40*142,857, or $57,143. Thus, of the $200,000 in the traditional IRA, $142,857 is rolled-over and $57,143 is paid in taxes. (Note: $142,857 = $200,000/1.4) But if the tax is a gross tax imposed on both the amount rolled-over and the amount used to pay the roll-over taxes, then $120,000 is rolled-over and $80,000 is used to pay the taxes.

3.3.5.2.1 Computation Assuming a Net Tax: The taxpayer's Roth IRA investment of $142,857 will grow to $142,857(1.07)^{10}, or $281,021. In addition, the taxpayer's $80,000 savings will compound at 7% per year but taxes will be due on the growth at the end of the 10-year period. Accordingly, the after-tax accumulation of the savings account will equal $80,000(1.07)^{10}*(0.60) + 0.40*$80,000, or $94,423 +

$32,000, or $126,423. Therefore, the total accumulation is $281,021 + $126,423, or $407,444. *Note that this is better than rolling over the entire $200,000 and paying the taxes out of the non-pension savings*. Why?

3.3.5.2.2 Computation Assuming a Gross Tax: The taxpayer's Roth IRA investment of $120,000 will grow to $120,000(1.07)^{10}, or $236,058. In addition, the taxpayer's $80,000 savings will compound at 7% per year but taxes will be due on the growth at the end of the 10-year period. Accordingly, the after-tax accumulation of the savings account will equal $80,000(1.07)^{10}*(0.60) + 0.40*$80,000, or $94,423 + $32,000, or $126,423. Therefore, the total accumulation is $236,058 + $126,423, or $362,481. This is the same accumulation as paying the roll-over taxes out of the savings account and rolling over the entire $200,000.

3.3.5.2.3 Sadly, the tax imposed on the conversion of a traditional IRA into a Roth IRA is the *gross tax* as discussed immediately above.

3.3.5.3 One why to describe the conversion is that it is the equivalent of (1) using IRA funds to pay the conversion tax (producing no net benefit if tax rates are constant) and then (2) redeploying external funds from some other investment into the Roth IRA. Accordingly, all of the benefit from the conversion is equivalent to moving funds from some other investment into a Roth IRA. If the funds are currently deployed in some investment inferior to a Roth IRA (which is the same as a traditional IRA), then the conversion will be beneficial.

3.3.5.4 More on Gross Taxes vs. Net Taxes: The income tax is a gross tax imposed on all your earnings rather than on only the portion you keep. The estate tax is a gross tax imposed on the gross value of a decedent's taxable estate but the gift tax is a net tax imposed only on the net transfer. While the estate tax and gift tax nominally use the same rates, because one is a gross tax and one is a net tax, the common rate schedule is misleading. For example, suppose X has $10,000,000 and this amount can either be transferred as a lifetime gift or as a death-time devise, each taxed at 40%. If transferred as a gift, the taxpayer can transfer $7,140,000. But if held until death, the taxpayer's estate will incur a tax of $4,000,000, leaving only $6,000,000 for the decedent's beneficiaries.

3.4 Discussion Questions (p. 3-19)

3.4.1 Question 1: (1) Whether the investment is deductible when made, (2) whether the returns are taxed periodically, at disposition or never, and (3) the applicable rate of tax.

3.4.2 Question 3: The SPDA benefits from deferral of tax while the money market account does not. Accordingly, the SPDA benefits more than the money market account from high interest rates because a greater amount of tax is deferred. In addition, the SPDA benefits from a longer investment horizon because the longer the invest horizon, the greater the amount of deferral.

3.4.3 Question 6: Taxpayers may choose to save through a money market account to maintain liquidity. While it is virtually costless to withdraw funds from a money market account, there typically are penalties for early withdrawal of pension or insurance funds, and pension funds cannot be used as collateral for a loan or else the tax-exempt feature of the pension trust will be lost. Insurance contracts include an insurance component. Some investors may not want to pay for the insurance component. In addition, early termination of an insurance contract may incur extra fees.

3.4.4 Question 10: This election might not make sense if the taxpayer expects his tax rate to increase in the next three years. The taxpayer has to compare the tax paid today with the present value of the tax to be paid over the 4 years. Whichever present value is lower is the favored alternative. More formally, let V denote the gain on rollover, and t_1, t_2, t_3, t_4 denote the taxpayer's tax rate in periods 1, 2, 3, and 4, respectively. The taxpayer can invest at an after-tax rate of r. We assume the taxpayer is making the decision at the end of the first period, period 1. Thus, include entire amount of gain, V, in the current period if $Vt_1 < [(V/4)t_1] + [(V/4)t_2]/(1+r) + [(V/4)t_3]/(1+r)^2 + [(V/4)t_4]/(1+r)^3$

3.5 Exercises (p. 3-19)

3.5.1 Exercise 11:

3.5.1.1 Part (a); For the money market fund: $\$1{,}000[1 + 0.08(1 – 0.25)]^{20} = \$3{,}207.14$. For the land investment, $\$1{,}000(1 + 0.07)^{20}(1 – 0.25) + 0.25(\$1{,}000) = \$2{,}902.26 + 250.00 = \$3{,}152.26$.

3.5.1.2 Part (b): For the money-market fund, the internal rate of return is $(3{,}207.14/1{,}000)^{1/20} – 1$, or 6%. For the land investment, the internal rate of return is $(\$3{,}152.26/\$1{,}000)^{1/20} – 1$, or 5.91%.

3.5.2 Exercise 12:

3.5.2.1 Part (a): $\$5{,}000(1 + 0.08)^{30}(1 – 0.30) + 0.30(\$5{,}000)$, or \$35,219 + \$1,500, or \$36,719. The internal rate of return is $(\$36{,}719/\$5{,}000)^{1/30} – 1$, or 6.86%.

3.5.2.2 Part (b): $\$5{,}000(1 + 0.08)^{30}(1 – 0.20) + 0.20(\$5{,}000)$, or \$40,251 + \$1,000, or \$41,251. The internal rate of return is $(\$41{,}251/\$5{,}000)^{1/30} – 1$, or 7.28%.

3.5.3 Exercise 15:[1] If the money is put into a in a traditional IRA, the accumulation will be $[(\$1{,}000)/(1 – 0.28)](1 + 0.10)^{40}(1 – 0.28)$, or \$45,259. If the money is put into

[1] Note that this answer assumes that the taxpayer pays the income taxes of \$280 out of the taxpayer's regular salary. If we assume that the income taxes are paid out of the bonus, then the taxpayer invests \$1,000 in the

a Roth IRA, the accumulation will be $1,000(1 + 0.10)^{40}$, or $45,259. Because the after-tax contributions can be equalized, there is no expected difference. Change the investment horizon to two years, and assume that the client believes the investment will not grow at all in the first year and then will grow by 20% in the second year. Now, what are the accumulations? If $1,000(1 – 0.28) is placed in a traditional IRA, then it will equal $1,389 after one year (no growth) and $1,667 after year 2, so the taxes will equal $467, leaving $1,200 after taxes. If placed into a Roth IRA, the after-tax return also will equal $1,200.

3.5.4 Exercise 16: The accumulation in the traditional IRA becomes $[\$1,000/(1 – 0.28)](1 + 0.10)^{40}(1 – 0.20)$, or $50,288.06. The Roth IRA accumulation does not change (because it is independent of future tax rates). Now, the traditional IRA is the better investment.

3.5.5 Exercise 19: If the money is left in the traditional IRA, the after-tax accumulation will be $\$20,000(1.12)^{40}(0.76)$, or $1,414,375. If the money is placed into a Roth IRA, it will yield $\$20,000(1.12)^{40}$, or $1,861,019. However, we must account for the opportunity cost of the roll-over, namely losing 0.24 times $20,000 (that is, $4,800). If that amount had been placed in a type II vehicle, it would grow to an after-tax amount of $\$4,800(1.12)^{40}(0.76)$ + $4,800*0.24, or $339,450 + $1,152, or $340,602. Accordingly, the net accumulation of the roll-over is $1,861,019 - $340,602, or $1,520,417. This dominates the non-roll-over strategy by $106,042. This answer assumes that taxes on the roll-over would be invested in a type II (SPDA) vehicle if not used to pay taxes on the roll-over.

3.5.6 Exercise 20: If R is the annual rate of return, V is the amount available for roll-over, t_0 is the tax rate at the time of the roll-over decision and t_n is the tax rate at the time the funds are removed from the investment, then leaving the funds in the traditional IRA yields $V(1 + R)^n(1 – t_n)$. If the investment is rolled over and (gross) taxes are paid on the roll-over, then the Roth IRA will yield $V(1 – t_0)(1 + R)^n$. If $t_0 = t_n$, these amounts are equal.

3.5.7 Exercise 22: Remember that electing to roll-over from a traditional IRA to a Roth IRA does not change the pre-tax investment in any way; it only changes the way taxes are paid. If the initial roll-over is not reversed, then the taxpayer owes taxes at a rate of 39.6% on $100,000, or $39,600. If the roll-over is reversed and then redone, the taxpayer will owe taxes of 39.6% on only $80,000, or $31,680. That is, the client saves 39.6% on $20,000, or $7,920. This amount presumably can be invested in a type II investment. Note that the taxpayer's retirement funds take an immediate 20% hit regardless of what is done with respect to the roll-over: reversing a roll-over does not put money back into your pension account. The future investment return, R, does not change the calculation, though the greater R, the more the client will see the $7,920 on the savings.

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traditional IRA and $720 in the Roth IRA. The computations change but the equality of investment returns remains true.